TRINITY COLLEGE

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS SPECIALIST UNITS 1&2

If required by your examination administrator, please place your student identification label in this box

Section Two: Calculator-assumed

WA student number: In figures | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	49	35
Section Two: Calculator-assumed	12	12	100	94	65
				Total	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

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Section Two: Calculator-assumed **65% (94 Marks)**

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

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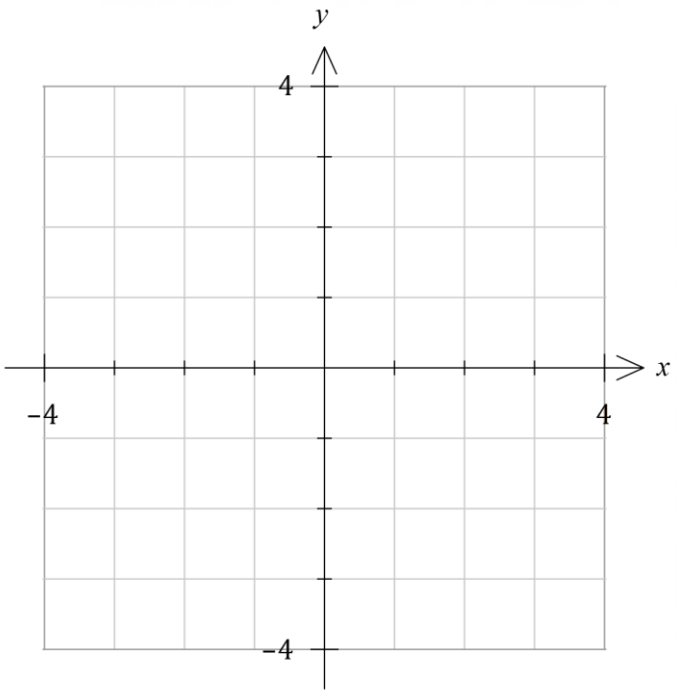
Question 8 **(9 marks)**

Triangle ABC has vertices $A(1,0), B(3,0)$ and $C(3,-1)$.

(a) Draw triangle ABC on the axes below, labelling all vertices. (1 mark)



Triangle ABC is transformed by matrix $\mathbf{T}_1 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ to form triangle $A'B'C'$.

(b) Determine the coordinates of A', B' and C' and draw labelled triangle $A'B'C'$ on the axis above. (2 marks)

(c) Describe the geometric transformation that matrix $\mathbf{T}_1$ represents. (2 marks)

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Triangle $A'B'C'$ is rotated $180°$ about the origin to form triangle $A''B''C''$.

(d) Determine the transformation matrix $\mathbf{T}_2$ that represents the transformation from triangle $A'B'C'$ to triangle $A''B''C''$ and draw labelled triangle $A''B''C''$ on the same axis as triangle ABC. (2 marks)

(e) Determine the single matrix $\mathbf{T}_3$ that will transform triangle $A''B''C''$ to triangle ABC and describe the geometric transformation that $\mathbf{T}_3$ represents. (2 marks)

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Question 9 **(8 marks)**

(a) Determine the number of integers between 1 and 2022 inclusive that are divisible by 17 or divisible by 5. (2 marks)

(b) Codes such as ZSTZT and XZUTY are made by randomly selecting five letters from the last eight letters in the alphabet. Determine what fraction of all possible codes contain repeated letters. (3 marks)

(c) Determine the number of different ways that the digits in the number 38723945 can be arranged so that all the odd digits are next to each other. (3 marks)

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Question 10 **(8 marks)**

(a) Express a displacement of 75 m on a bearing of $311°$ in component form to one decimal place, given that the unit vectors **i** and **j** are directed due east and north respectively. (2 marks)

(b) Express the velocity $\begin{pmatrix} 97 \\ -78 \end{pmatrix}$ m/s as a linear combination of the velocities $\begin{pmatrix} 5 \\ -2 \end{pmatrix}$ m/s and $\begin{pmatrix} -3 \\ 4 \end{pmatrix}$ m/s. (3 marks)

(c) In the trapezium $OXYZ$, $\overrightarrow{OX} = \mathbf{x}$, $\overrightarrow{OZ} = \mathbf{z}$, $\overrightarrow{ZY} = 3\mathbf{x}$ and M and N are the midpoints of XY and YZ respectively. Determine the value of the constant μ and the value of the constant λ if $\overrightarrow{MN} = \mu\mathbf{x} + \lambda\mathbf{z}$. (3 marks)

DO NOT WRITE IN THIS AREA AS IT WILL BE CUT OFF

Question 11 **(8 marks)**

(a) A chord of length 20.8 cm is 15.3 cm away from the centre of a circle. Determine the distance of a chord of length 35.2 cm from the centre of the same circle. (3 marks)

(b) Chords PR and QS of the circle with centre O are perpendicular to each other and intersect at T. Determine, with reasoning, the distance OT when $RT = 7$ cm, $ST = 14$ cm and $QS = 16$ cm. (5 marks)

Question 12 **(7 marks)**

A small body is oscillating vertically on a spring in a laboratory so that the height h cm of its centre above the floor t seconds after measurements began can be modelled by

$$h = a\cos\big(b(t-c)\big) + 75.$$

The maximum height of the body of 130 cm is first reached when $t = 1$ and again when $t = 9$.

(a) Determine the value of each of the positive constants a, b and c. If more than one value is possible, give the value closest to 0. (3 marks)

The ceiling is 2.2 m above the floor.

(b) If d cm is the distance from the centre of the body to the ceiling, determine the relationship between d and t as a sine function. (2 marks)

(c) At what time is the centre of the body first equidistant from the ground and the ceiling? (2 marks)

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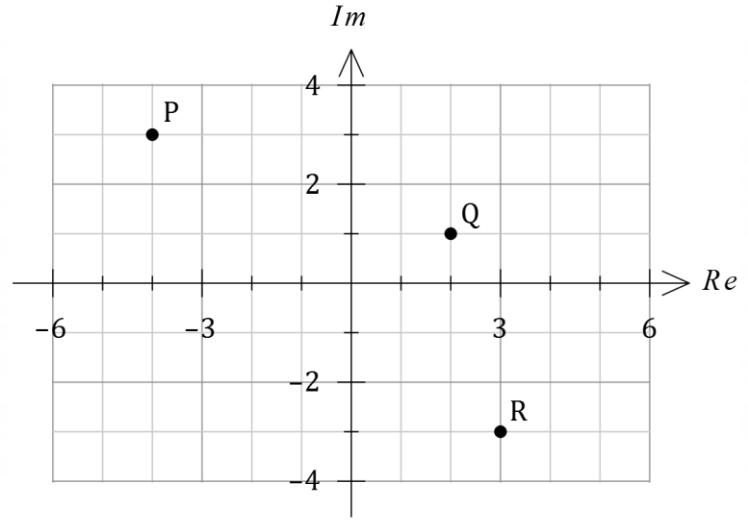
Question 13 **(8 marks)**

The points P, Q and R shown in the complex plane below represent the complex numbers z_1, z_2 and z_3 respectively.



(a) On the same diagram plot the points A, B and C to represent the following complex numbers. (3 marks)

$$A: z_1 + 2 - 5i, \qquad B: z_3 - z_2, \qquad C: \overline{z_3}.$$

(b) Determine

(i) $\text{Re}((z_2)^2)$. (1 mark)

(ii) $\text{Im}\left(z_3 \div \left(1 - \sqrt{2}i\right)\right)$. (1 mark)

(c) Given that the complex number z_1 is a solution to the equation $z^2 + bz + c = 0$, determine the value of the real constant b and the real constant c. (3 marks)

See next page

Question 14 **(7 marks)**

(a) Prove that $\sin(\theta + \phi) - \sin(\theta - \phi) = 2\cos\theta\sin\phi$. (2 marks)

(b) Express the difference $\sin 7x - \sin x$ as a product in the form $k\sin\alpha\cos\beta$. (1 mark)

(c) Determine all exact solutions of the equation $\sin 7x - \cos 4x - \sin x = 0$ for $0 \leq x \leq \frac{\pi}{2}$, justifying your answer. (4 marks)

See next page

Question 15 **(8 marks)**

(a) Two forces are given by $\mathbf{F}_1 = \begin{pmatrix} 33 \\ 16.5 \end{pmatrix}$ N and $\mathbf{F}_2 = \begin{pmatrix} -12 \\ 9 \end{pmatrix}$ N. Determine

(i) the component of $\mathbf{F}_1$ that is acting parallel to $\mathbf{F}_2$. (2 marks)

(ii) the component of $\mathbf{F}_1$ that is acting perpendicular to $\mathbf{F}_2$. (2 marks)

(b) A small body, acted on by three horizontal forces, is in equilibrium on a smooth level table. One force has magnitude 55 N and acts on a bearing of $245°$. Determine the magnitudes of the other two forces given that they act on bearings of $105°$ and $350°$. (4 marks)

Question 16 **(6 marks)**

(a) Balls are taken at random and without replacement from a bag that contains 6 pink, 8 yellow, 11 green, 12 silver, 13 blue and 15 red balls. Demonstrate use of the pigeon-hole principle to determine the least number of balls that should be taken from the bag to be certain that at least 10 of them are of the same colour. (3 marks)

(b) A school social committee of 4 people is to be selected from a group of 11 secondary students, 7 primary students and 12 teachers. Determine how many of the possible committees have more primary students than teachers. (3 marks)

See next page

Question 17 **(8 marks)**

A small plane that has a cruising speed of 221 km/h is planning to fly from airport A to airport B, where $\overrightarrow{AB} = \begin{pmatrix} -429 \\ 363 \end{pmatrix}$ km and a steady wind of $\begin{pmatrix} 15 \\ -8 \end{pmatrix}$ km/h is forecast for the local region.

Let the velocity vector that the small plane should set for the flight be $\begin{pmatrix} p \\ q \end{pmatrix}$ km/h.

(a) Explain why

(i) $\sqrt{p^2 + q^2} = 221$. (1 mark)

(ii) $p + 15 = -429k$ and $q - 8 = 363k$, where k is a constant. (2 marks)

(b) Determine another relationship between p and q, besides $\sqrt{p^2 + q^2} = 221$. (2 marks)

(c) Determine the velocity vector $\begin{pmatrix} p \\ q \end{pmatrix}$ and hence calculate the expected flight time for the small plane from A to B. (3 marks)

Question 18 **(9 marks)**

Points A, B and C lie on a circle and join to form an isosceles triangle in which $CA = CB$. Let E be a point on the base AB so that CE extended meets the circle at D.

(a) Sketch a diagram to show the above information. (1 mark)

(b) Use angle theorems to prove that triangle AED is similar to triangle CEB. (3 marks)

(c) Show that $AD \times BE = DE \times CB$. (1 mark)

See next page

(d) Prove that $\frac{1}{AD}+\frac{1}{BD}=\frac{1}{DE}\times\frac{AB}{CB}$. (4 marks)

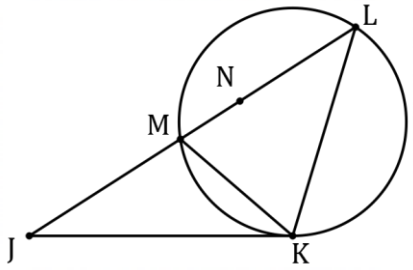
See next page

Question 19 **(8 marks)**

(a) The points A, B, C and D lie in order on a semi-circle with centre O and diameter AOD. When $\angle DOC = 68°$ and $\angle BCO = 55°$, determine the size of $\angle OAB$. (3 marks)

(b) In the diagram, JK is the tangent to the circle at K and JML is a secant that cuts the circle at M and L.

Point N lies between M and L so that $JK = JN$.



(i) Prove that $\angle JMK = \angle JKL$. (2 marks)

(ii) Prove that KN bisects $\angle MKL$. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____